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# An Enhanced Pseudo-Differential Operator Associated with Bessel Operator

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## ABSTRACT

*In this paper a pseudo-differential operator $P'(x, D)$ in terms of a symbol having broad range of values than the operators defined previously and the inverse Hankel transform of the symbol is defined. The boundedness of the pseudo-differential operator in certain Sobolev-type space with Hankel transform is also established.*

## Keywords

Pseudo-differential Operator, Bessel Operator, Sobolev-type space, Convolution product, Hankel-type transformation.

## Subject Classification

Mathematics Subject Classification: 44A15, 46F.

## 1. INTRODUCTION

The Hankel-type transformation of $\varphi \in L^1(I), I = (0, \infty)$ is defined by

$$(H_\mu \varphi)(x) = \int_0^\infty (xy)^{-\mu} J_\mu(xy)\varphi(y) y^{\mu/2} dy, x \epsilon I \qquad (1)$$

where $(xy)^{-\mu} J_\mu(xy) y^{\mu/2}$ represents the kernel of this transformation, as usual, $J_\mu$ is the Bessel function of the first kind and order $\mu$. We shall assume that $\mu \geq -1/2$. Since $(x)^{-\mu} J_\mu(x)$ is bounded on I, the Hankel-type transformation $(H_\mu \varphi)(x)$ is bounded on I provided $\int_0^\infty x^{\mu/2} |\varphi(x)| dx < \infty$.

Also we get, $(H_\mu \varphi)(0) = \frac{1}{2^\mu \Gamma(\mu+1)} \int_0^\infty \varphi(y) y^{\mu/2} dy$.
The inversion formula for (1) is given by

$$\varphi(x) = \int_0^\infty (xy)^{-\mu} J_\mu(xy) H_\mu \varphi(y) y^{\mu/2} dy, x \epsilon I \qquad (2).$$

Altenberg [5] introduced the space $\mathcal{H}$ consisting of all infinitely differentiable functions $\varphi$ defined on I= $(0,\infty)$, such that for all $m, k \epsilon \mathbb{N}_0$ the quantities

$$\gamma_{m,k} = sup_{x \epsilon I} (1 + x^2)^m |(x^{-1} d/dx)^k \varphi(x)| < \infty.$$

Zaidman [12] studied a class of pseudo-differential operators (p.d.o's) using Schwartz's theory of Fourier transformation. Pseudo-differential operators associated to a numerical valued symbol $a(x, y)$ were discussed by Pathak and Prasad [10], Singh and Prasad [2]. In the investigation of the pseudo-differential operator $P(x, D)$ depending on the transformation $H_\mu$ it assumes that the symbol $a(x, y)$ posses derivatives which satisfy certain growth conditions, as follows:

$$H_{\mu,a} \varphi(x) = \int_0^\infty (xy)^{-\mu} J_\mu(xy) a(x, y) H_\mu \varphi(y) y^{\mu/2} dy, x \epsilon I$$

Where $(H_\mu \varphi)(x) = \int_0^\infty (xy)^{-\mu} J_\mu(xy) \varphi(y) y^{\mu/2} dy, x \epsilon I.$

From [8] the symbol $a(x, y)$ is defined to be the complex valued infinitely differentiable functions on $I\times I$ which satisfy

$\left|(x^{-1}D_x)^\alpha (y^{-1}D_y)^\beta a(x, y)\right| \leq C^{\alpha+\beta+1} \alpha! \beta! (1 + y)^{m-\beta}, \forall \alpha, \beta \in \mathbb{N}_0$ and $m$ is a fixed real number. The class of all such symbols is defined by $\mathcal{H}^m$. From [10] we know that for any $\varphi, \psi \in \mathcal{H}$

$$(x^{-1}D_x)^k (\varphi\psi) = \sum_{\upsilon=0}^{k} \binom{k}{\upsilon} (x^{-1}D_x)^\upsilon \varphi (x^{-1}D_x)^{k-\upsilon} \psi.$$

In this paper we have used the Hankel transformation defined by (1) to develop a theory of pseudo-differential operator associated with Bessel operator corresponding to [2,11].

## 2. HE HANKEL CONVOLUTION

We use the following results on Hankel convolution in the sequel of $\Delta(x, y, z)$ from Zemanian [1]. Let $\Delta(x, y, z)$ be the area of triangle with sides $x, y, z$ if such a triangle exists. For $\mu > 0$, set

$$D(x, y, z) = 2^{3\mu-1}(\pi)^{-1/2}[\Gamma(\mu + 1)^2] \times (\Gamma(\mu + 1/2)^{-1})(xyz)^{-2\mu}[\Delta(x, y, z)]^{2\mu-1}$$

If $\Delta$ exists and zero otherwise. We note that $D(x, y, z) \geq 0$ and that $D(x, y, z)$ is symmetric in $x, y, z$ and we have $\int_0^\infty j(zt) D(x, y, z) d\mu(z) = j(xt) j(yt)$
where

$$d\mu(z) = \left[2^{\mu/2}\Gamma(\mu + 1)\right]^{-1} z^{\mu/2} dz$$

and

$$j(x) = 2^{\mu/2}\Gamma(\mu + 1) x^{-\mu} J_\mu(x).$$