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The convolution product associated with the Bessel type wavelet transform

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ABSTRACT

In this paper the convolution product associated with the Bessel type wavelet transformation is investigated. Certain norm inequalities for the convolution product are established.

Introduction

Hankel convolution has been studied by many authors in recent past. Following Cholewinski [1], Haimo [2], Hirschman Jr. [3], the Hankel type convolution for the following form of the Hankel type transformation of a function $f \in L^1_\sigma(I)$, where $I = (0,\infty)$ and

$$L^1_\sigma(I) = \left\{ f: \int_0^\infty |f(x)|\, d\sigma(x) < \infty \{, \qquad I = (0,\infty) \right\}.$$

Namely,

$$\left(h_{\alpha,\beta} f\right)(x) = \tilde{f}(x) = \int_0^\infty j_{\alpha-\beta}(xt)\, f(t)\, d\sigma(t)\,, \tag{1.1}$$

where

$$j_{\alpha-\beta}(x) = 2^{-2\beta}\, \Gamma(2\alpha)\, x^{2\beta}\, J_{-2\beta}(x) \; and \; J_\lambda(x)$$

is the Bessel function of first kind and of order λ. Here

$$d\sigma(t) = \frac{t^{2(\alpha-\beta)}}{2^{-2\beta}\, \Gamma(2\alpha)}\, dt.$$

We say that $f \in L^p_\sigma(I),\ 1 \le p < \infty,\ if$

$$\|f\|_{p,\sigma} = \left(\int_0^\infty |f(x)|^p\, d\sigma(x) \right)^{\frac{1}{p}} < \infty.$$

If $f \in L^1_\sigma(I)$ *and* $h_{\alpha,\beta} f \in L^1_\sigma(I)$ then the inverse Hankel type transform is given by

$$f(x) = \left(h^{-1}_{\alpha,\beta}[\tilde{f}]\right)(x) = \int_0^\infty j_{\alpha-\beta}(xt)\left(h_{\alpha,\beta} f\right)(t)\, d\sigma(t) \tag{1.2}$$

If $f \in L^1_\sigma(I)$, $g \in L^1_\sigma(I)$ then the Hankel type convolution is defined by

$$(f \# g)(x) = \int_0^\infty (\tau_x f)(y)\, g(y)\, d\sigma(y)\,, \tag{1.3}$$

where the Hankel type translation τ_x is given by

$$(\tau_x f)(y) = \tilde{f}(x,y) = \int_0^\infty D(x,y,z)\, f(x)\, d\sigma(z), \tag{1.4}$$

where

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