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N – dimensional generalized heat equation and its heat polynomial

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ABSTRACT
In this paper we consider the generalized heat equation of n^{th} order

$$\frac{\partial^2 u}{\partial r^2}+\frac{n-1}{r}\frac{\partial u}{\partial r}-\frac{d^2}{r^2}u=\frac{\partial u}{\partial t}.$$

If the initial temperature is an even power function, then the heat transform with the source solution as the kernel gives the heat polynomials. We discuss various properties of the heat polynomial and its Appell type transform. Also, we give series representation of the heat transform when the initial temperature is a power function.

Introduction

In this paper we shall establish various properties of the polynomial solutions and its Appell transforms of the generalized heat equation of the n^{th} order.

$$\frac{\partial^2 u}{\partial r^2}+\frac{n-1}{r}\frac{\partial u}{\partial r}-\frac{d^2}{r^2}u=\frac{\partial u}{\partial t}\ ,$$

where $r^2 = x_1^2 + x_2^2 + \dots\dots + x_n^2$. We shall also give a series expansion of the generalized temperature in terms of Laguerre polynomials and confluent hypergeometric functions. Most of the results derived here are similar to the ones found in [6, 7], which are for the less general equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{4a}{x}\frac{\partial u}{\partial x}=\frac{\partial u}{\partial t}$$

which in turn is a generalization of the ordinary heat equation [8]

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}.$$

These known results can be considered as special cases of our more general results when $d = 0$ *and* $n = 1$.

Preliminaries:

Consider the equation

$$\Delta_n\,\psi\,(r,\theta)=\frac{\partial\psi}{\partial t}\qquad ,$$

where $r^2 = x_1^2 + x_2^2 + \cdots \dots . + x_n^2$ and $\theta = tan^{-1}\left(\frac{r}{x_n}\right)$. Then we have

$$\frac{\partial^2\psi}{\partial r^2}+\frac{n-1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2\,sin^{n-2}\theta}\frac{\partial}{\partial\theta}\left[sin^{n-2}\theta\,\frac{\partial\psi}{\partial\theta}\right]=\frac{\partial\psi}{\partial t}.$$

If the solution is of the type

$$\psi(r,\theta)=u(r,t)\,p(\theta),$$

then

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