Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 90 (2016) 37360-37369

ISSN: 2229-712X

Solutions of the Generalized Heat Equation and its Integral Representations

B.B.Waphare
MAEER's MIT Arts, Commerce and Science College, Alandi, Pune, India.

ARTICLE INFO

Article history:
Received: 20 November 2015;
Received in revised form:
25 December 2015;
Accepted: 31 December 2015;

Keywords
Integral Representations,
Hankel Type Transform,
Appelltype Transform,
Bessel Type Function;
Poisson-Hankel-Stieljes
Transform.

ABSTRACT
In this paper we have explored the problem for generalized temperature functions considered over positive and negative time. We have established representation theorems and their applications.

1. Introduction

In recent past, characterizations for generalized temperature functions defined for positive time which may be represented by Poisson-Hankel-Stieltjesintegral transforms were derived in the papers of Cholewinski and Haimo [1], Haimo [3].Our aim in the present paper is to explore the problem for generalized temperature functions considered over negative time. Although the representation theorems obtained can be proved by techniques analogous to those of the previous results, we use the more elegant approach of appealing to the Appell transform to reduce these cases to those dealt with earlier. In addition, we investigate some other generalized temperature functions which have integral representations.

2.Notations and terminology:

The generalized heat equation is

$\Delta_x u(x,t) = \frac{\partial}{\partial t} u(x,t)$, where (2.1)

$\Delta_x f(x) = f''(x) + \frac{2(\alpha-\beta)+1}{x} f'(x)$, where $\left(\alpha-\beta+\frac{1}{2}\right)$ is a fixed positive number.

A generalized temperature function is a function of class C^2 which satisfies the generalized heat equation. We denote the class of such functions by H.

The fundamental solution of the generalized heat equation is the function

$G(x,y;t) = (1/2t)^{\alpha-\beta+1} e^{-(x^2+y^2)/4t} g(xy/2t)$ (2.2)

where $g(z) = 2^{\alpha-\beta}\Gamma(\alpha-\beta+1) z^{-(\alpha-\beta)} I_{\alpha-\beta}(z)$, $I_\lambda(z)$ is the Bessel function of imaginary argument of order λ.We write $G(x;t)$ for $G(x, 0; t)$.

If $V(x,t)$ is an arbitrary function of two variables, then its Appell type transform $V^A(x,t)$ is given by

$V^A(x,t) = V^A_{x,t}(x,t) = G(x;t)\, V(x/t, -1/t)$. (2.3)

Now we define a subclass H^* of H which plays an important role in our theory:

A generalized temperature function $u(x,t)$ is a member of H^* for $a < t < b$, if and only if, for every $t, t', a < t' < t < b$,

$u(x,t) = \int_0^\infty G(x,y;t-t')\, u(y,t')\, d\mu(y)$, (2.4)

where

$$d\mu(x) = \frac{2^{-(\alpha-\beta)}}{\Gamma(\alpha-\beta+1)} x^{2(\alpha-\beta)+1}\, dx,$$

The integral converging absolutely. Functions in $H^*$are said to have the Huygens property.

By Theorem 6.4 of [3], functions in H^*have a complex integral representation as well. Infact, we have the following result.

Tele:
E-mail addresses: balasahebwaphare@gmail.com