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The Range of the Hankel type and Extended Hankel type Transformations

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ABSTRACT
In this paper we have studied the range of Hankel type and extended Hankeltype transforms on some spaces of functions. Further the Paley-Wiener type theorem for the Hankel type transforms is also established.

1. Introduction

In view of [27], we define Hankel type transform as

$$f(x) = (\mathcal{H}_{\alpha,\beta}\, g)(x) = \int_0^\infty (xy)^{\alpha+\beta} J_{\alpha-\beta}(xy)\, g(y)\, dy,\ \ x \in R_+ = (0,\infty) \qquad (1.1)$$

if the integral converges in some sense (absolutely, improper, or mean convergence), where $J_{\alpha-\beta}(x)$is the Bessel type function of the first kind [1]. According to [27] if $Re(\alpha-\beta) > -1$, then the Hankel type transform is an automorphism of $L_2(R_+)$ and its inverse on $L_2(R_+)$ has the symmetric form

$$g(x) = (\mathcal{H}_{\alpha,\beta} f)(x) = \int_0^\infty (xy)^{\alpha+\beta} J_{\alpha-\beta}(xy)\, f(y)\, dy, x \in R_+,\ Re(\alpha-\beta) > -1\,\cdot \qquad (1.2)$$

Following [13,22], we define the extended Hankel type transform as

$$f(x) = (\mathcal{H}_{\alpha,\beta}\, g)(x) = \int_0^\infty (xy)^{\alpha+\beta} J_{\alpha-\beta,m}(xy)\, g(y)\, dy, \qquad (1.3)$$

$Re(\alpha-\beta) < -1,\ Re(\alpha-\beta) \neq -3,-5,\dots\dots, x \in R_+,\ 1-2m > Re(\alpha-\beta) > -2m-1,\ m > 0$,where$J_{\alpha-\beta,m}(x)$ is the truncated (or "cut") Bessel type function of the first kind and is defined as

$$J_{\alpha-\beta,m}(x) = J_{\alpha-\beta}(x) - \sum_{k=0}^{m-1} \frac{(-1)^k \left(\frac{x}{2}\right)^{\alpha-\beta+2k}}{\Gamma(3\alpha+\beta+k)\, k!}, \qquad (1.4)$$

where ,$1-2m > Re(\alpha-\beta) > -2m-1, m \geq 0$

and the integral is understood in L_2 sense. The extended Hankel type transform (1.3) is a bounded operator in $L_2(R_+)$ and its inverse, also a bounded operator in $L_2(R_+)$, has been proved to have the form (see [12])

$$g(x) = -x^{-2\beta} D_x x^{2\beta} \int_0^\infty (xy)^{\alpha+\beta} J_{-\alpha-3\beta,m+1}(xy)\, f(y)\, dy, \qquad (1.5)$$

$x \in R_+,\ 1-2m > Re(\alpha-\beta) > -2m-1,\ m > 0,\ D_x \equiv \frac{d}{dx}$.

Formula (1.5) can be rewritten in the equivalent form, symmetric to formula (1.3). In fact, if we put

$$f_N(x) = \begin{cases} f(x), & x \in [1/N,\ N] \\ 0, & otherwise \end{cases} \qquad (1.6)$$

Then $f_N(x)$ tends to $f(x)$ in $L_2(R_+)$ norm. Therefore if $g_N(x)$ is the inverse of the extended Hankel type transform (1.5) of $f_N(x)$, then $g_N(x)$tends to $g(x)$ in $L_2(R_+)$norm. By using the relation

$$\frac{d}{dx}\left(x^{\alpha+3\beta} J_{-\alpha-3\beta,m+1}(x)\right) = -x^{\alpha-\beta} J_{\alpha-\beta,m}(x),\ Re(\alpha-\beta) > -2m-1, m \geq 0, \qquad (1.7)$$

we have

$$g_N(x) = -x^{-2\beta} \frac{d}{dx} x^{2\beta} \int_{1/N}^{N} (xy)^{\alpha+\beta} J_{-\alpha-3\beta,m+1}(xy)\, f(y)\, dy$$

$$= \int_{1/N}^{N} (xy)^{\alpha+\beta} J_{\alpha-\beta,m}(xy)\, f(y)\, dy\,. \qquad (1.8)$$

Therefore,

$$g(x) = (\mathcal{H}_{\alpha,\beta} f)(x) = \int_0^\infty (xy)^{\alpha+\beta} J_{\alpha-\beta,m}(xy)\, f(y)\, dy, \qquad (1.9)$$

$1-2m > Re(\alpha-\beta) > -2m-1, m > 0$, wherethe integral is understood in L_2 sense.

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