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A Generalized Finite Hankel Type Transformation and a Parseval Type Equation

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Abstract: *In this paper, we study the finite Hankel type transformation on spaces of generalized functions by developing new procedure. Two Hankel type integral transformations $h_{\alpha,\beta}$ and $h^*_{\alpha,\beta}$ are considered and they satisfy Parseval type equation defined by (1.2). We have defined a space $S_{\alpha,\beta}$ of functions and a space $L_{\alpha,\beta}$ of complex sequences and it is further shown that $h^*_{\alpha,\beta}$ and $h'_{\alpha,\beta}$ are isomorphisms from $S_{\alpha,\beta}$ onto $L_{\alpha,\beta}$ and $S'_{\alpha,\beta}$ onto $L'_{\alpha,\beta}$ respectively. Finally some applications of new generalized finite Hankel type transformation are established*

Keywords: *Finite Hankel type transformation, Parseval type equation, generalized finite Hankel type transformation.*

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1. Introduction

I.N. Sneddon [14] was first to introduce finite Hankel transforms of classical functions. The same was later studied by [3], [4], [7], [15]. Recently Zemanian [18], Pandey and Pathak [11] and Pathak [12] extended these transforms to certain spaces of distributions as a special case of general theory on orthonromal series expansions of generalized functions. Dube [5], Pathak and Singh [13] and Mendez and Negrin [10] investigated finite Hankel transformations in other spaces of distributions through a procedure quite different from that one which was developed in [18] and [12].

We define finite Hankel type transformation of the first kind by

$$\left(h_{\alpha,\beta} f\right)(n) = \int_0^1 x \; J_{\alpha-\beta}\,(\lambda_n\, x)\, f(x)\, dx, \;\; n = 0,1,2, \dots \dots .$$

for $(\alpha-\beta) \geq -\frac{1}{2}$, where J_v denotes the Bessel function of the first kind and order v and λ_n, $n = 0,1,2, \dots \dots$, represent the positive roots of $J_{\alpha-\beta}(x) = 0$ arranged in ascending order of magnitude [17, p.596].

For $(\alpha-\beta) \geq -\frac{1}{2}$ and $a \geq \frac{1}{2}$, we introduce the space $U_{\alpha,\beta,a}$ of finitely differentiable functions on (0,1) such that

$$\rho_k^{\alpha,\beta,a}(\phi) = Sup_{0<x<1}\left|x^{a-1}\, B^{*k}_{\alpha,\beta}\, \phi(x)\right| < \infty, \text{ for every } k \in \mathbb{N},$$

where $B^*_{\alpha,\beta} = x^{-(\alpha-\beta)} D\, x^{4\alpha}\, D\, x^{-(3\alpha+\beta)}$.

$U_{\alpha,\beta,a}$ is equipped with the topology generated by the family of seminorms $\left\{\rho_k^{\alpha,\beta,a}\right\}_{k=0}^{\infty}$. Thus $U_{\alpha,\beta,a}$ is a Frechet space. $U'_{\alpha,\beta,a}$ denotes the dual of $U_{\alpha,\beta,a}$ and is endowed with the weak topology.

For $f \in U'_{\alpha,\beta,a}$, the generalized finite Hankel type transform of f is defined by

$$F(n) = < f(x)\,, x\, J_{\alpha-\beta}\,(\lambda_n x) >, \;\; for \;\; n = 0,1,2, \dots \dots \qquad (1.1)$$

