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THE PSEUDO DIFFERENTIAL-TYPE OPERATOR $(-x^{-1}D)^{(\alpha-\beta)}$ ASSOCIATED WITH THE FOURIER-BESSEL TYPE SERIES REPRESENTATION

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Abstract. For certain Frechet space F consisting of complex valued C^{∞} functions defined on $I = (0, \infty)$ and characterized by their asymptotic behavior near the boundaries, we show that :

(i) The Pseudo differential operator $(-x^{-1}D)^{(\alpha-\beta)}, (\alpha-\beta) \in R, D = \frac{d}{dx}$, is an automorphism (in the topological sense) on F.

(ii) $(-x^{-1}D)^{(\alpha-\beta)}$ is almost an inverse of the Hankel type transform $h_{\alpha,\beta}$ in the sense that $h_{\alpha,\beta} \circ (x^{-1}D)^{(\alpha-\beta)}(\phi) = h_{\circ}(\phi)$,for all $\phi \in F$ and $(\alpha-\beta) \in R$

(iii) $(-x^{-1}D)^{(\alpha-\beta)}$ has a Fourier-Bessel type series representation on a subspace $F_b \subset F$ and also on its dual F_b'.

1 Introduction

The theory of pseudo differential operators has been developed by many researchers in India and abroad. In recent years pseudo differential operators involving Hankel transform, Hankel convolution, Bessel operators etc. has been studied by many mathematicians. It is the purpose of this paper to give the Fourier-Bessel type series representation of the pseudo differential type operator $(-x^{-1}D)^{(\alpha-\beta)}$.

We denote by F the space of all C^{∞}- complex valued functions $\phi(x)$ defined on $I = (0, \infty)$, such that

$$\phi(x) = \sum_{i=0}^{k} a_i x^{2i} + O(x^{2k}) \tag{1.1}$$

near the origin and is rapidly decreasing as $x \to \infty$.

For $(\alpha-\beta) > -1/2$, we define a $(\alpha-\beta)^{th}$ order Hankel-type transform $h_{\alpha,\beta}$ on F by

$$\Phi(y) = [h_{\alpha,\beta}\phi(x)](y) = \int_0^{\infty} \phi(x)\mathcal{J}_{\alpha,\beta}(xy)dm(x) \tag{1.2}$$

where

$$dm(x) = m'(x)dx = \left[2^{\alpha-\beta}\Gamma(3\alpha+\beta)\right]^{-1} x^{4\alpha}dx,$$

$$\mathcal{J}_{\alpha,\beta}(x) = 2^{\alpha-\beta}\Gamma(3\alpha+\beta)x^{-(\alpha-\beta)}J_{\alpha-\beta}(x),$$